

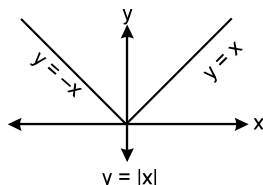


Fundamentals of Mathematics-II

He is unworthy of the name of man who is ignorant of the fact that the diagonal of square is incommensurable with its sidePlato

Absolute value function / modulus function :

The symbol of modulus function is $f(x) = |x|$ and is defined as: $y = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$



Properties of modulus : For any $a, b \in \mathbb{R}$

- (i) $|a| \geq 0$
- (ii) $|a| = |-a|$
- (iii) $|a| \geq a, |a| \geq -a$
- (iv) $|ab| = |a| |b|$
- (v) $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$
- (vi) $|a + b| \leq |a| + |b|$; Equality holds when $ab \geq 0$
- (vii) $|a - b| \geq ||a| - |b||$; Equality holds when $ab \geq 0$

Example # 1 : Solve the following linear equations

(i) $x|x| = 4$ (ii) $|x - 3| + 2|x + 1| = 4$

Solution :

(i) $x|x| = 4$

If $x > 0$

$\therefore x^2 = 4 \Rightarrow x = \pm 2$

$\therefore x = 2$ ($\because x \geq 0$)

If $x < 0 \Rightarrow -x^2 = 4 \Rightarrow x^2 = -4$ which is not possible

(ii) $|x - 3| + 2|x + 1| = 4$

case I : If $x \leq -1$

$\therefore -(x - 3) - 2(x + 1) = 4$

$\Rightarrow -x + 3 - 2x - 2 = 4 \Rightarrow -3x + 1 = 4$

$\Rightarrow -3x = 3 \Rightarrow x = -1$

case II : If $-1 < x \leq 3$

$\therefore -(x - 3) + 2(x + 1) = 4$

$\Rightarrow -x + 3 + 2x + 2 = 4 \Rightarrow x = -1$ which is not possible

case III : If $x > 3$

$x - 3 + 2(x + 1) = 4$

$3x - 1 = 4 \Rightarrow x = 5/3$ which is not possible $\therefore x = -1$ Ans.

Rational function :

A rational function is a function of the form, $y = f(x) = \frac{g(x)}{h(x)}$, where $g(x)$ & $h(x)$ are polynomial functions.

Irrational function :

An irrational function is a function $y = f(x)$ in which the operations of addition, subtraction, multiplication, division and raising to a fractional power are used.

For example $y = \frac{x^3 + x^{1/3}}{2x + \sqrt{x}}$ is an irrational function

(a) The equation $\sqrt{f(x)} = g(x)$, is equivalent to the following system

$f(x) = g^2(x)$ & $g(x) \geq 0$



- (b) The inequation $\sqrt{f(x)} < g(x)$, is equivalent to the following system
 $f(x) < g^2(x)$ & $f(x) \geq 0$ & $g(x) \geq 0$
- (c) The inequation $\sqrt{f(x)} > g(x)$, is equivalent to the following system
 $g(x) \leq 0$ & $f(x) \geq 0$ or $g(x) \geq 0$ & $f(x) > g^2(x)$

Example # 2 : Solve : $x + 2 > 2\sqrt{1-x^2}$

Solution : $4(1-x^2) < (x+2)^2$ and $x+2 \geq 0$ & $1-x^2 \geq 0$

$$x \in \left(-\infty, -\frac{4}{5}\right) \cup (0, \infty) \quad \dots(1)$$

$$x \in [-2, \infty) \quad \dots(2)$$

$$x \in [-1, 1] \quad \dots(3)$$

$$(1) \cap (2) \cap (3)$$

$$\left[-1, -\frac{4}{5}\right) \cup (0, 1]$$

Self Practice Problem :

(1) $\sqrt{2x^2 + x - 6} < x$

(2) $\sqrt{5-x} > x+1$

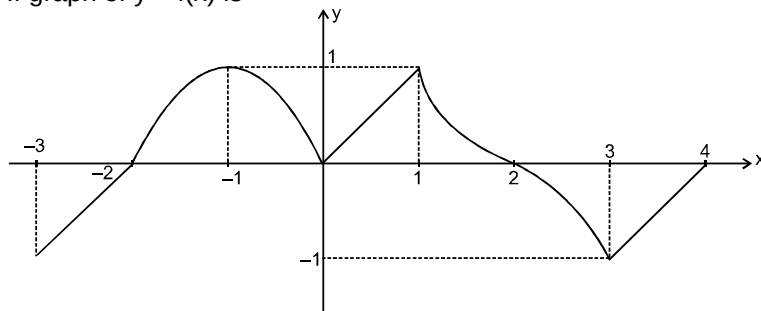
(3) $x+3 + \sqrt{x^2+4x-5} > 0$

(4) $\sqrt{x} - \sqrt{4-x} \geq 1$

Ans. (1) $\left[\frac{3}{2}, 2\right)$ (2) $(-\infty, 1)$ (3) $(-\infty, -1] \cup [5, \infty)$ (4) $\left[\frac{4+\sqrt{7}}{2}, 4\right]$

Graphs Related to modulus :

If graph of $y = f(x)$ is



then draw graph of

(a) $y = -f(x)$

(b) $y = f(-x)$

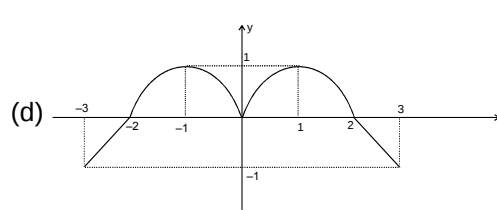
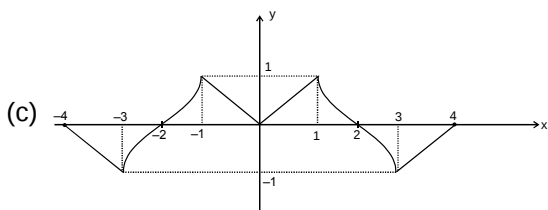
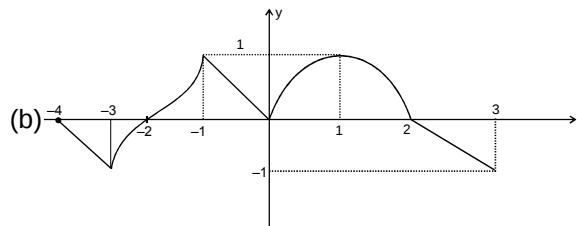
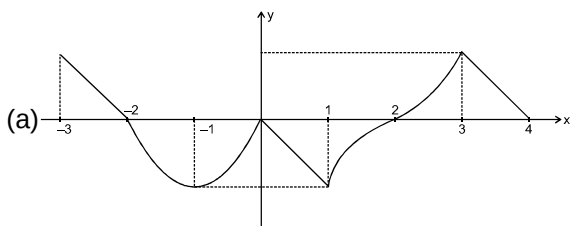
(c) $y = f(|x|)$

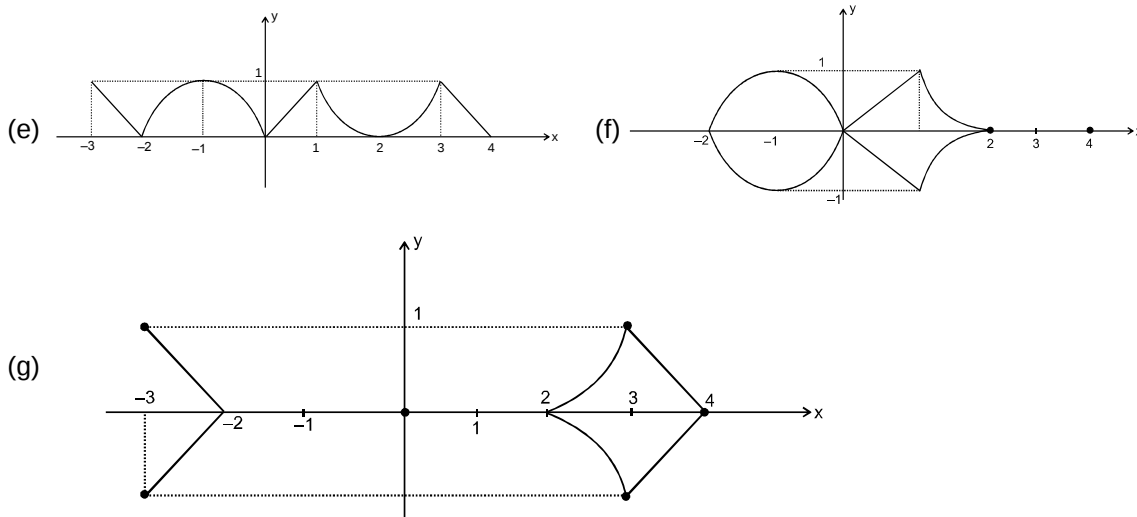
(d) $y = f(-|x|)$

(e) $y = |f(x)|$

(f) $|y| = f(x)$

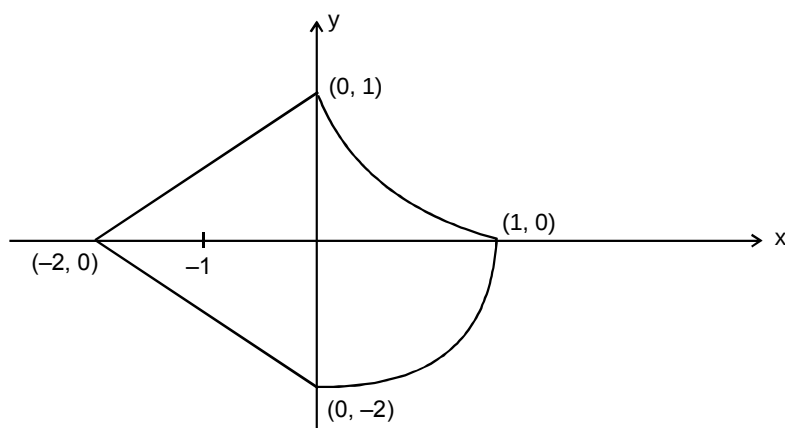
(g) $|y| = -f(x)$



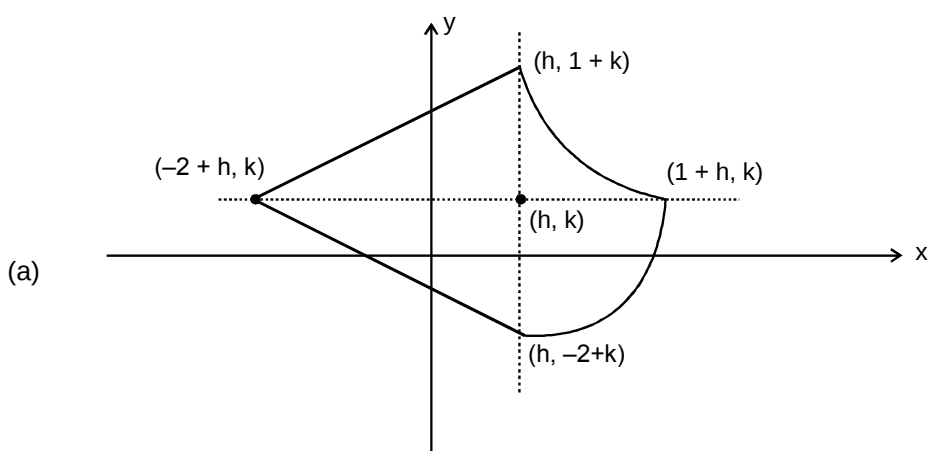


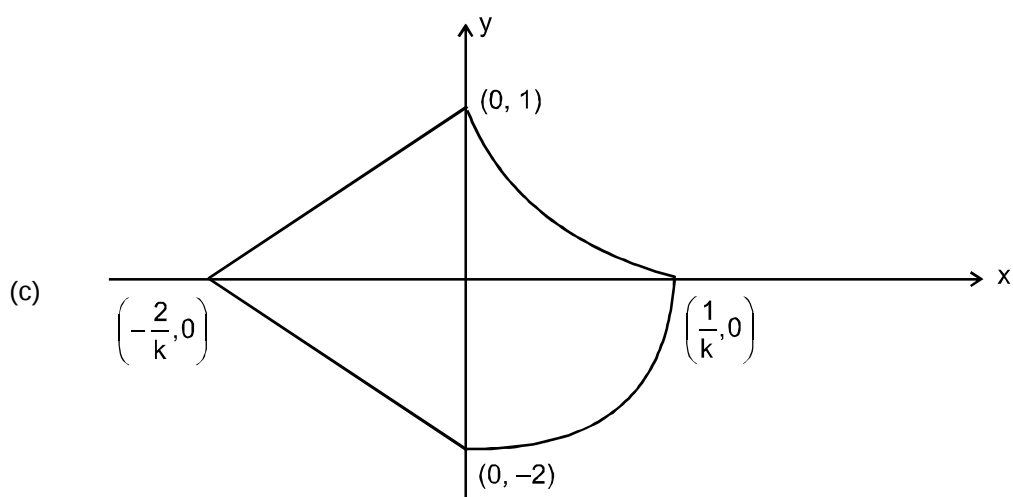
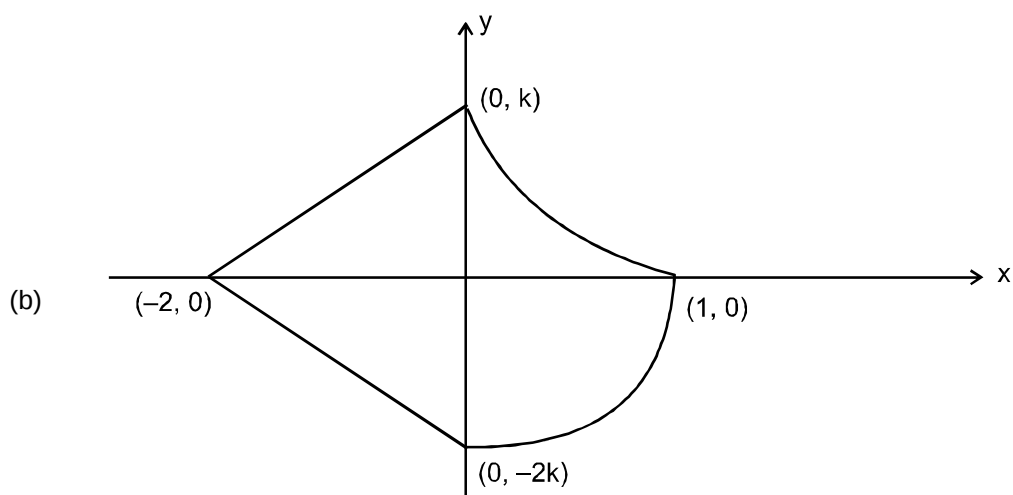
Graphical Trasformation :

If graph of $y = f(x)$ is



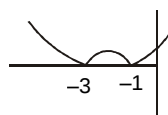
then graph of (a) $y - k = f(x - h)$ (b) $y = kf(x)$, ($k > 0$) (c) $y = f(kx)$, ($k > 0$)





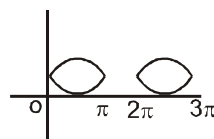
Example # 3 : $y = |x^2 + 4x + 3|$

Solution :



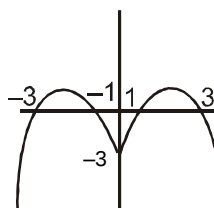
Example # 4 : $|y-1| = \sin x$

Solution :



Example # 5 : $y = -x^2 + 4|x| - 3$

Solution :



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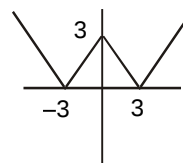
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Example # 6 : $y = ||x| - 3|$

Solution :

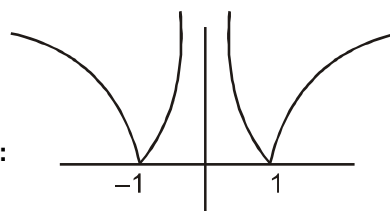


Example # 7 : $y = \sin\left(\frac{x}{3}\right)$

Solution : period is 6π

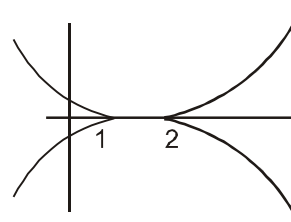
Example # 8 : $y = |\sin x - 3|$

Solution : Graphical Transformation



Example # 9 : $y = |-\ln|-x||$

Solution :



Example # 10 : $|y| = x^2 - 3x + 2$

Solution :





Exercise-1

Marked questions are recommended for Revision.

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Modulus Function & Equation

A-1. Write the following expression in appropriate intervals so that they are bereft of modulus sign

- (i) $|x^2 - 7x + 10|$ (ii) $|x^3 - x|$ (iii) $|2^x - 2|$
 (iv) $|x^2 - 6x + 10|$ (v) $|x - 1| + |x^2 - 3x + 2|$ (vi) $\sqrt{x^2 - 6x + 9}$
 (vii) $2^{(x-1)} + |x + 2| - 3^{[x+1]}$

A-2. Draw the labeled graph of following

- (i) $y = |7 - 2x|$ (ii) $y = |x - 1| - |3x - 2|$
 (iii) $y = |x - 1| + |x - 4| + |x - 7|$ (iv) $y = |4x + 5|$ (v) $y = |2x - 3|$

A-3. Solve the following equations

- (i) $|x| + 2|x - 6| = 12$ (ii) $||x + 3| - 5| = 2$
 (iii) $||x - 2| - 2| - 2| = 2$ (iv) $|4x + 3| + |3x - 4| = 12$

A-4. Solve the following equations :

- (i) $x^2 - 7|x| - 8 = 0$ (ii) $|x^2 - x + 1| = |x^2 - x - 1|$
 (iii) $|x^3 - 6x^2 + 11x - 6| = 6$ (iv) $|x^2 - 2x| + x = 6$
 (v) $|x^2 - x - 6| = x + 2$

A-5. Find the number of real roots of the equation

- (i) $|x|^2 - 3|x| + 2 = 0$ (ii) $||x - 1| - 5| = 2$ (iii) $|2x^2 + x - 1| = |x^2 + 4x + 1|$

A-6. Find the sum of solutions of the following equations :

- (i) $x^2 - 5|x| - 4 = 0$ (ii) $|x|^3 - 15x^2 - 8|x| - 11 = 0$
 (iii) $(x - 3)^2 + |x - 3| - 11 = 0$ (iv) $2^{|x|} + 3^{|x|} + 4^{|x|} = 9$
 (v) $||x - 3| - 4| = 1$

A-7. Find number of solutions of the following equations

- (i) $|x - 1| + |x - 2| + |x - 3| = 9$ (ii) $|x - 1| + |x - 2| + |x - 3| + |x - 4| = 4$
 (iii) $|x| + |x + 2| + |x - 2| = p, p \in \mathbb{R}$

A-8. Find the minimum value of $f(x) = |x - 1| + |x - 2| + |x - 3|$

A-9. If $x^2 - |x - 3| - 3 = 0$, then $|x|$ can be

A-10. If $|x^3 - 6x^2 + 11x - 6|$ is a prime number then find the number of possible integral values of x .

Section (B) : Modulus Inequalities

B-1. Solve the following inequalities :

- (i) $|x - 3| \geq 2$ (ii) $|x - 2| - 3 \leq 0$ (iii) $||3x - 9| + 2| > 2$
 (iv) $|2x - 3| - |x| \leq 3$ (v) $|x - 1| + |x + 2| \geq 3$ (vi) $||x - 1| - 1| \leq 1$



B-2. Solve the following inequalities :

$$(i) \left| 1 + \frac{3}{x} \right| > 2 \quad (ii) \left| \frac{3x}{x^2 - 4} \right| \leq 1 \quad (iii) \frac{|x+3| + x}{x+2} > 1$$

$$(iv) |x^2 + 3x| + x^2 - 2 \geq 0 \quad (v) |x+3| > |2x-1|$$

B-3. Solve the following inequalities

$$(i) |x^3 - 1| \geq 1 - x \quad (ii) |x^2 - 4x + 4| \geq 1 \quad (iii) \frac{|x+2| - x}{x} < 2$$

$$(iv) \frac{|x-2|}{x-2} > 0 \quad (v) |x-2| > |2x-3| \quad (vi) |x+2| + |x-3| < |2x+1|$$

B-4. Solve the following equations

$$(i) |x^3 + x^2 + x + 1| = |x^3 + 1| + |x^2 + x|$$

$$(ii) |x^2 - 4x + 3| + |x^2 - 6x + 8| = |2x - 5|$$

$$(iii) |x^2 + x + 2| - |x^2 - x + 1| = |2x + 1|$$

$$(iv) |x^2 - 2x - 8| + |x^2 + x - 2| = 3|x + 2|$$

$$(v) |2x - 3| + |x + 5| \leq |x - 8|$$

B-5. Find the solution set of the inequalities $|x^2 + x - 2| \leq 0$ and $|x^2 - x + 2| \geq 0$

Section (C) : Miscellaneous Modulus Equations & Inequalities

C-1. Write the following expression in appropriate intervals so that they are bereft of modulus sign

$$(i) |\log_{10} x| + |2^{x-1} - 1| \quad (ii) |(\log_2 x)^2 - 3(\log_2 x) + 2| \quad (iii) |5^{x^2-4x+5} - 25|$$

C-2. Solve the equations $\log_{100} |x + y| = 1/2$, $\log_{10} y - \log_{10} |x| = \log_{100} 4$ for x and y .

C-3. Solve the inequality

$$(i) (\log_2 x)^2 - |(\log_2 x) - 2| \geq 0$$

$$(ii) 2|\log_3 x| + \log_3 x \geq 3$$

$$(iii) \text{ Find the complete solution set of } 2^x + 2^{|x|} \geq 2\sqrt{2}$$

C-4. Find the number of real solution(s) of the equation $|x-3|^{3x^2-10x+3} = 1$

C-5. If x, y are integral solutions of $2x^2 - 3xy - 2y^2 = 7$, then find the value of $|x + y|$

C-6. If $x, |x+1|, |x-1|$ are three terms of an A.P., then find the number of possible values of x

Section (D) : Irrational Inequalities

D-1. Solve the following inequalities :

$$(i) \frac{\sqrt{2x-1}}{x-2} < 1 \quad (ii) x - \sqrt{1-|x|} < 0 \quad (iii) \sqrt{x^2 - x - 6} < 2x - 3$$

$$(iv) \sqrt{x^2 - 6x + 8} \leq \sqrt{x+1} \quad (v) \sqrt{x^2 - 7x + 10} + 9 \log_4 \left(\frac{x}{8} \right) \geq 2x + \sqrt{14x - 20 - 2x^2} - 13$$

$$(vi) x - 3 < \sqrt{x^2 + 4x - 5} \quad (vii) \sqrt{x^2 - 5x - 24} > x + 2 \quad (viii) \sqrt{4-x^2} \geq \frac{1}{x}$$

$$(ix) \frac{\sqrt{x+7}}{x+1} > \sqrt{3-x}$$

D-2. Solve the equation $\sqrt{a(2^x - 2)} + 1 = 1 - 2^x$ for every value of the parameter a .





Section (E) : Transformation of curves

E-1. Draw the graph of followings —

(i) $y = -|x + 2|$

(ii) $y = ||x - 1| - 2|$

(iii) $y = |x + 2| + |x - 3|$

(iv) $|y| + x = -1$

E-2. Draw the graphs of the following curves :

(i) $y = -\frac{1}{|2x + 1|}$

(ii) $\frac{y}{|x| - 1} = -1$

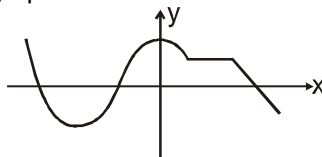
(iii) $|y - 3| = |x - 1|$

(iv) $y = \frac{|x^2 - 1|}{(x^2 - 1)} \ln x$

E-3. Draw the graph of $y = \log_{1/2}(1 - x)$.

E-4. Find the set of values of λ for which the equation $|x^2 - 4x| - 12 = \lambda$ has 6 distinct real roots.

E-5. If $y = f(x)$ has following graph



Then draw the graph of

(i) $y = |f(x)|$

(ii) $y = f(|x|)$

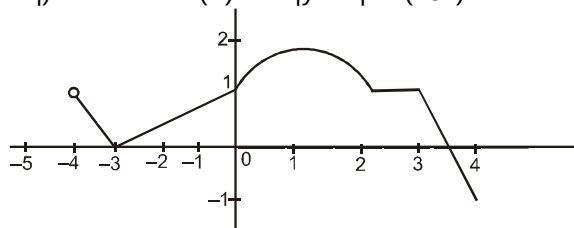
(iii) $y = f(-|x|)$

(iv) $y = |f(|x|)|$

E-6. If $y = f(x)$ is shown in figure given below, then plots the graph for

(A) $y = f(|x + 2|)$

(B) $|y - 2| = f(-3x)$



E-7. Find the number of roots of equation

(i) $3^{|x|} - |2 - |x|| = 1$

(ii) $x + 1 = x \cdot 2^x$

E-8. Find values of k for which the equation $|x^2 - 1| + x = k$ has

(i) 4 solution

(ii) 3 solutions

(iii) 1 solution

(iv) 2 solutions

Exercise-2

Marked questions are recommended for Revision.

* Marked Questions may have more than one correct option.

1. Number of integral values of 'x' satisfying the equation $3^{|x+1|} - 2 \cdot 3^x = 2 \cdot |3^x - 1| + 1$ are

(A) 1

(B) 2

(C) 3

(D) 4

2. $|x^2 + 6x + p| = x^2 + 6x + p \forall x \in \mathbb{R}$ where p is a prime number then least possible value p is

(A) 7

(B) 11

(C) 5

(D) 13

3. If $(\log_{10} x)^2 - 4|\log_{10} x| + 3 = 0$, the product of roots of the equation is :

(A) 3

(B) 10^4

(C) 10^8

(D) 1

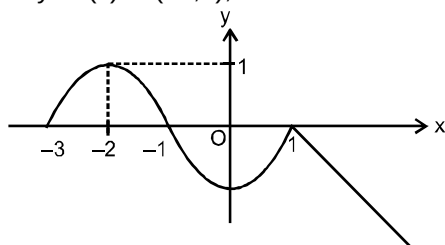




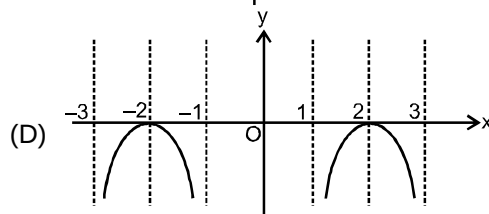
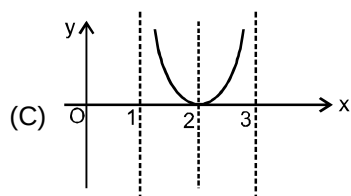
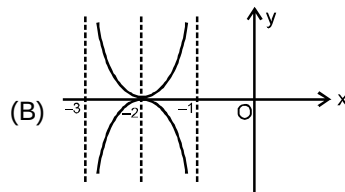
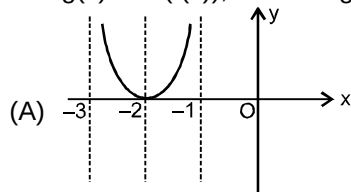
4. The equation $||x - 1| + a| = 4$ can have real solutions for x if a belongs to the interval
 (A) $(-\infty, 4]$ (B) $(4, \infty)$ (C) $(-4, \infty)$ (D) $(-\infty, -4) \cup (4, \infty)$
5. The number of values of x satisfying the equation $|2x + 3| + |2x - 3| = 4x + 6$, is
 (A) 1 (B) 2 (C) 3 (D) 4
6. Number of prime numbers satisfying the inequality $\log_3 \frac{|x^2 - 4x| + 3}{x^2 + |x - 5|} \geq 0$ is equal to
 (A) 1 (B) 2 (C) 3 (D) 4
7. If $|x + 2| + y = 5$ and $x - |y| = 1$ then the value of $(x + y)$ is
 (A) 1 (B) 2 (C) 3 (D) 4
8. The number of value of x satisfying the equation $|x - 1|^A = (x - 1)^7$, where $A = \log_3 x^2 - 2 \log_x 9$
 (A) 1 (B) 2 (C) 0 (D) 3
9. The number of integral value of x satisfying the equation $|\log_{\sqrt{3}} x - 2| - |\log_3 x - 2| = 2$
 (A) 1 (B) 2 (C) 3 (D) 4
10. The sum of all possible integral solutions of equation $||x^2 - 6x + 5| - |2x^2 - 3x + 1|| = 3|x^2 - 3x + 2|$ is
 (A) 10 (B) 12 (C) 13 (D) 15
11. The complete solution set of the inequality $(|x - 1| - 3)(|x + 2| - 5) < 0$ is $(a, b) \cup (c, d)$ then the value of $|a| + |b| + |c| + |d|$ is
 (A) 14 (B) 15 (C) 16 (D) 17
12. The product of all the integers which do not belong to the solution set of the inequality $\left| \frac{3|x| - 2}{|x| - 1} \right| \geq 2$ is
 (A) -1 (B) -4 (C) 4 (D) 0
13. Let $f(x) = |x - 2|$ and $g(x) = |3 - x|$ and
 A be the number of real solutions of the equation $f(x) = g(x)$
 B be the minimum value of $h(x) = f(x) + g(x)$
 C be the area of triangle formed by $f(x) = |x - 2|$, $g(x) = |3 - x|$ and x -axis and $\alpha < \gamma < \beta < \delta$ where $\alpha < \beta$ are the roots of $f(x) = 4$ and $\gamma < \delta$ are the roots of $g(x) = 4$, then the value of sum of digits of $\frac{\alpha^2 + \beta^2 + \gamma^2 + \delta^2}{ABC}$.
 (A) 7 (B) 8 (C) 11 (D) 9
- 14*. If $f(x) = |x + 1| - 2|x - 1|$ then
 (A) maximum value of $f(x)$ is 2. (B) there are two solutions of $f(x) = 1$.
 (C) there is one solution of $f(x) = 2$. (D) there are two solutions of $f(x) = 3$.
- 15*. The solution set of inequality $|x| < \frac{a}{x}$, $a \in \mathbb{R}$, is
 (A) $(-\sqrt{-a}, 0)$ if $a < 0$ (B) $(0, \sqrt{a})$ if $a > 0$ (C) $\emptyset$ if $a = 0$ (D) $(0, a)$ if $a > 0$
- 16*. If a and b are the solutions of equation : $\log_5 \left(\log_{64} |x| - \frac{1}{2} + 25^x \right) = 2x$, then
 (A) $a + b = 0$ (B) $a^2 + b^2 = 128$ (C) $ab = 64$ (D) $a - b = 8$



17. The number of solution of the equation $\log_3|x-1| \cdot \log_4|x-1| \cdot \log_5|x-1| = \log_5|x-1| + \log_3|x-1| \cdot \log_4|x-1|$ are
 (A) 3 (B) 4 (C) 5 (D) 6
18. Find the number of all the integral solutions of the inequality $\frac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x^2-9)} \leq 0$
 (A) 1 (B) 2 (C) 3 (D) 4
19. Find the complete solution set of the inequality $\frac{1-\sqrt{21-4x-x^2}}{x+1} \geq 0$
 (A) $[2\sqrt{6}-2, 3]$ (B) $[-2, -2\sqrt{6}, -1]$
 (C) $[-2-2\sqrt{6}, -1] \cup [2\sqrt{6}-2, 3]$ (D) $[-2, -2\sqrt{6}, -1] \cup [2\sqrt{6}-2, 3]$
20. The solution set of the inequality $\frac{|x+2|-|x|}{\sqrt{4-x^3}} \geq 0$ is
 (A) $[-1, \sqrt[3]{4})$ (B) $[1, \sqrt[3]{4})$ (C) $[-1, \sqrt[3]{2})$ (D) $[0, \sqrt[3]{4})$
21. The number of integers satisfying the inequality $\sqrt{\log_{1/2}^2 x + 4\log_2 \sqrt{x}} < \sqrt{2} (4 - \log_{16} x^4)$ are
 (A) 2 (B) 3 (C) 4 (D) 5
22. If $f_1(x) = ||x| - 2|$ and $f_n(x) = |f_{n-1}(x) - 2|$ for all $n \geq 2, n \in \mathbb{N}$, then number of solution of the equation $f_{2015}(x) = 2$ is
 (A) 2015 (B) 2016 (C) 2017 (D) 2018
23. If graph of $y = f(x)$ in $(-3, 1)$, is as shown in the following figure



and $g(x) = \ln(f(x))$, then the graph of $y = g(-|x|)$ is





24*. Solution set of inequality $||x| - 2| \leq 3 - |x|$ consists of :

- (A) exactly four integers (B) exactly five integers
(C) Two prime natural number (D) One prime natural number

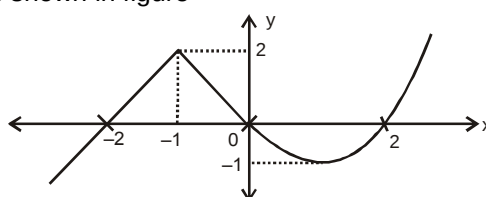
25*. If $a \neq 0$, then the inequation $|x - a| + |x + a| < b$

- (A) has no solutions if $b \leq 2|a|$ (B) has a solution set $\left(\frac{-b}{2}, \frac{b}{2}\right)$ if $b > 2|a|$
(C) has a solution set $\left(\frac{-b}{2}, \frac{b}{2}\right)$ if $b < 2|a|$ (D) All above

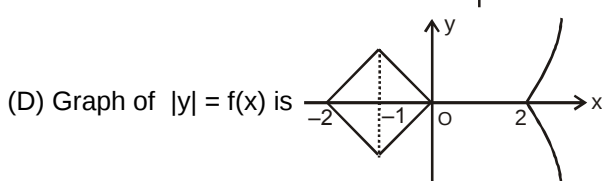
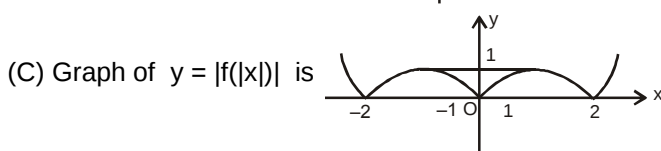
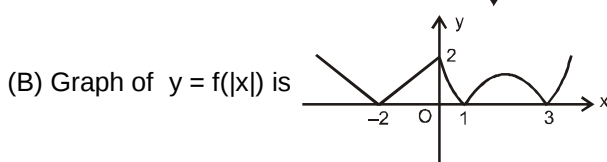
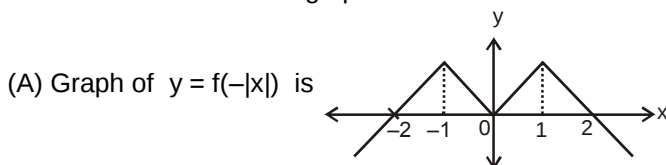
26*. The equation $||x - a| - b| = c$ has four distinct real roots, then

- (A) $a > b - c > 0$ (B) $c > b > 0$
(C) $a > c + b > 0$ (D) $b > c > 0$

27*. If graph of $y = f(x)$ is as shown in figure



then which of the following options is/are correct ?



28*. Consider the equation $|x^2 - 4|x| + 3| = p$

- (A) for $p = 2$ the equation has four solutions
(B) for $p = 2$ the equation has eight solutions
(C) there exists only one real value of p for which the equation has odd number of solutions
(D) sum of roots of the equation is zero irrespective of value of p





- 29*. Consider the equation $|\ln x| + x = 2$, then
 (A) The equation has two solutions (B) Both solutions are positive
 (C) One root exceeds one and other is less than one (D) Both roots exceed one
- 30*. Consider the equation $||x - 1| - |x + 2|| = p$. Let p_1 be the value of p for which the equation has exactly one solution. Also p_2 is the value of p for which the equation has infinite solution. Let α be the sum of all the integral values of p for which this equation has solution then
 (A) $p_1 = 0$ (B) $p_2 = 3$ (C) $\alpha = 6$ (D) $p_1 + p_2 = 4$
31. Number of the solution of the equation $2^x = |x - 1| + |x + 1|$ is
 (A) 0 (B) 1 (C) 2 (D) ∞
32. Number of the solution of the equation $x^2 = |x - 2| + |x + 2| - 1$ is
 (A) 0 (B) 3 (C) 2 (D) 4
33. $f(x)$ is polynomial of degree 5 with leading coefficient = 1, $f(4) = 0$. If the curve $y = |f(x)|$ and $y = f(|x|)$ are same, then the value of $f(5)$ is
 (A) 405 (B) -405 (C) 45 (D) -45
34. The area bounded by the curve $y \geq |x - 2|$ and $y \leq 4 - |x - 3|$ is
 (A) $\frac{13}{2}$ (B) 7 (C) $\frac{15}{2}$ (D) 8





Exercise-3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

Marked questions are recommended for Revision.

* Marked Questions may have more than one correct option.

- Draw the graph of $y = |x|^{1/2}$ for $-1 \leq x \leq 1$.
- The number of real solutions of the equation $|x|^2 - 3|x| + 2 = 0$ is :
(A) 4 (B) 1 (C) 3 (D) 2
3. If p, q, r are any real numbers, then
(A) $\max(p, q) < \max(p, q, r)$ (B) $\min(p, q) = \frac{1}{2}(p + q - |p - q|)$
(C) $\max(p, q) < \min(p, q, r)$ (D) None of these
- Let $f(x) = |x - 1|$. Then
(A) $f(x^2) = (f(x))^2$ (B) $f(x + y) = f(x) + f(y)$ (C) $f(|x|) = |f(x)|$ (D) None of these
- If x satisfies $|x - 1| + |x - 2| + |x - 3| \geq 6$, then
(A) $0 \leq x \leq 4$ (B) $x \leq -2$ or $x \geq 4$ (C) $x \leq 0$ or $x \geq 4$ (D) None of these
- Solve $|x^2 + 4x + 3| + 2x + 5 = 0$.
- If p, q, r are positive and are in A.P., then roots of the quadratic equation $px^2 + qx + r = 0$ are real for
(A) $\left| \frac{r}{p} - 7 \right| \geq 4\sqrt{3}$ (B) $\left| \frac{r}{p} - 7 \right| < 4\sqrt{3}$
(C) all p and r (D) no p and r
- The function $f(x) = |ax - b| + c|x| \forall x \in (-\infty, \infty)$, where $a > 0, b > 0, c > 0$, assumes its minimum value only at one point if
(A) $a \neq b$ (B) $a \neq c$ (C) $b \neq c$ (D) $a = b = c$
9. Find the set of all solutions of the equation $2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1$
- The sum of all the real roots of the equation $|x - 2|^2 + |x - 2| - 2 = 0$ is _____.
- If α & β ($\alpha < \beta$) are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then
(A) $0 < \alpha < \beta$ (B) $\alpha < 0 < \beta < |\alpha|$ (C) $\alpha < \beta < 0$ (D) $\alpha < 0 < |\alpha| < \beta$
- If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that $\min f(x) > \max g(x)$, then the relation between b and c , is
(A) no relation (B) $0 < c < b/2$ (C) $|c| < \sqrt{2} |b|$ (D) $|c| > \sqrt{2} |b|$

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

- Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$
(1) is always positive (2) is always negative (3) does not exist (4) none of these
- The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is
(1) 3 (2) 2 (3) 4 (4) 1



3. The sum of the roots of the equation, $x^2 + |2x - 3| - 4 = 0$, is :
 (1) $-\sqrt{2}$ (2) $\sqrt{2}$ (3) -2 (4) 2
4. The equation $\sqrt{3x^2 + x + 5} = x - 3$, where x is real, has :
 (1) exactly four solutions (2) exactly one solutions
 (3) exactly two solutions (4) no solution
5. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is :
 (1) $(-\infty, \infty)$ (2) $(0, \infty)$ (3) $(-\infty, 0)$ (4) $(-\infty, \infty) - \{0\}$
6. If x is a solution of the equation, $\sqrt{2x+1} - \sqrt{2x-1} = 1$, $\left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2 - 1}$ is equal to
 (1) 2 (2) $\frac{3}{4}$ (3) $2\sqrt{2}$ (4) $\frac{1}{2}$
7. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in the A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is :
 (1) $\frac{\sqrt{34}}{9}$ (2) $\frac{2\sqrt{13}}{9}$ (3) $\frac{\sqrt{61}}{9}$ (4) $\frac{2\sqrt{17}}{9}$
8. Let $S = \{x \in \mathbb{R} : x \geq 0 \text{ and } 2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$. Then S :
 (1) contains exactly two elements. (2) contains exactly four elements.
 (3) is an empty set. (4) contains exactly one element



Answers

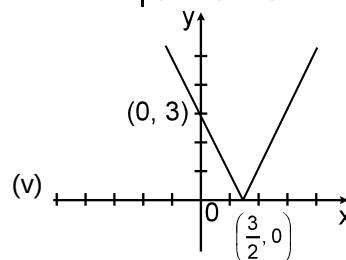
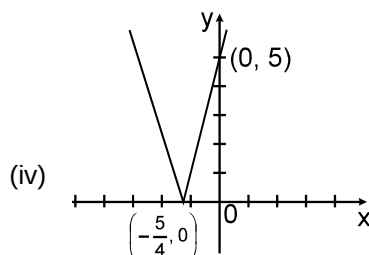
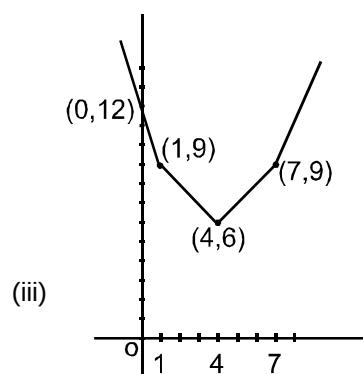
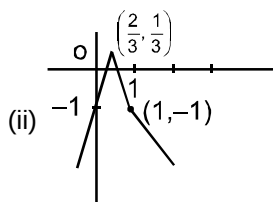
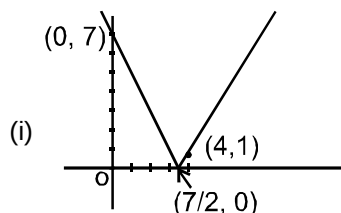
EXERCISE # 1

PART-I

Section (A) :

- A-1.** (i) $x^2 - 7x + 10, x > 5$ or $x \leq 2$; $-(x^2 - 7x + 10), 2 < x \leq 5$
 (ii) $x^3 - x, x \in [-1, 0] \cup [1, \infty)$; $x - x^3, x \in (-\infty, -1) \cup (0, 1)$
 (iii) $2^x - 2, x \geq 1$; $2 - 2^x, x < 1$ (iv) $x^2 - 6x + 10, x \in \mathbb{R}$
 (v) $x^2 - 2x + 1, x \geq 2$; $4x - x^2 - 3, 1 \leq x < 2$; $x^2 - 4x + 3, x < 1$
 (vi) $x - 3, x \geq 3$; $3 - x, x < 3$
 (vii) $2^{x-1} + x + 2 - 3^{x+1}, x \geq -1$; $2^{x-1} + x + 2 - 3^{-(x+1)}, -2 \leq x < -1$
 $2^{x-1} - x - 2 - 3^{-(x+1)}, x < -2$

A-2.



- A-3.** (i) $x = 0, 8$ (ii) $x = -10, -6, 0, 4$ (iii) $x = 0, \pm 4, 8$ (iv) $x = -\frac{11}{7}, \frac{13}{7}$
A-4. (i) ± 8 (ii) $0, 1$ (iii) $0, 4$ (iv) $-2, 3$ (v) $x \in \{-2, 2, 4\}$
A-5. (i) 4 (ii) 4 (iii) 4
A-6. (i) 0 (ii) 6 (iii) 0 (iv) 12 (v) 0
A-7. (i) 2 (ii) Infinite
 (iii) $p < 4$ no solution $p = 4$ one solution $p > 4$ Two solution
A-8. 2 **A-9.** $2, 3$ **A-10.** 0

Section (B) :

- B-1.** (i) $x \in (-\infty, 1] \cup [5, \infty)$ (ii) $x = 5$ or $x = -1$ (iii) $x \in \mathbb{R} - \{3\}$ (iv) $x \in [0, 6]$
 (v) $\mathbb{R}$ (vi) $[-1, 3]$





- B-2.** (i) $x \in (-1, 0) \cup (0, 3)$ (ii) $x \in (-\infty, -4] \cup [-1, 1] \cup [4, \infty)$
 (iii) $x \in (-5, -2) \cup (-1, \infty)$ (iv) $x \in \left(-\infty, -\frac{2}{3}\right] \cup \left[\frac{1}{2}, \infty\right)$ (v) $x \in \left(-\frac{2}{3}, 4\right)$
- B-3.** (i) $x \in (-\infty, -1] \cup [0, \infty)$ (ii) $x \in (-\infty, 1] \cup [3, \infty)$ (iii) $x \in (-\infty, 0) \cup (1, \infty)$
 (iv) $x \in (2, \infty)$ (v) $(1, 5/3)$ (vi) $(2, \infty)$
- B-4.** (i) $\{-1\} \cup [0, \infty)$ (ii) $[1, 2] \cup [3, 4]$ (iii) $x \in \left[-\frac{1}{2}, \infty\right)$
 (iv) $[1, 4] \cup \{-2\}$ (v) $\left[-5, \frac{3}{2}\right]$
- B-5.** $\{-2, 1\}$

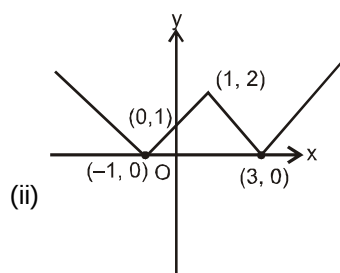
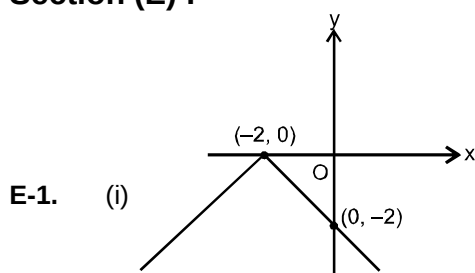
Section (C) :

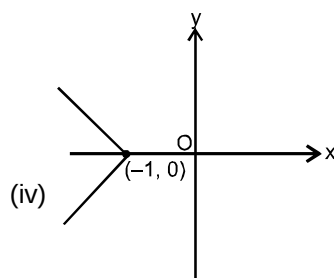
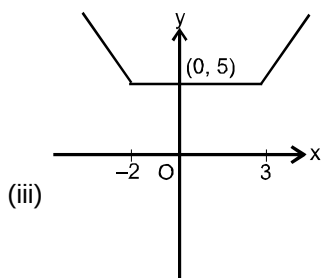
- C-1.** (i) $\log_{10}x + 2^{x-1} - 1$ $x \geq 1$
 $-(\log_{10}x + 2^{x-1} - 1)$ $0 < x < 1$
 (ii) $(\log_2x)^2 - 3(\log_2x) + 2$ $x \in (0, 2] \cup [4, \infty)$
 $-(\log_2x)^2 - 3(\log_2x) + 2$ $x \in (2, 4)$
 (iii) $5^{x^2-4x+5} - 25$ $x \in (-\infty, 1] \cup [3, \infty)$
 $25 - 5^{x^2-4x+5}$ $x \in (1, 3)$
- C-2.** $x = 10/3, y = 20/3$ & $x = -10, y = 20$
- C-3.** (i) $x \in \left(0, \frac{1}{4}\right] \cup [2, \infty)$ (ii) $\left(0, \frac{1}{27}\right] \cup [3, \infty)$ (iii) $(-\infty, \log_2(\sqrt{2}-1)] \cup \left[\frac{1}{2}, \infty\right)$
- C-4.** 3 **C-5.** 4 **C-6.** 2

Section (D) :

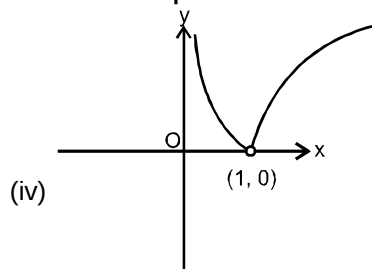
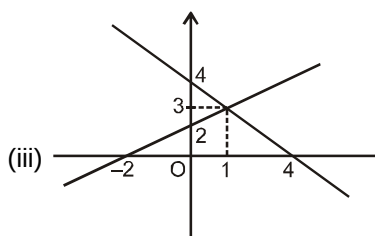
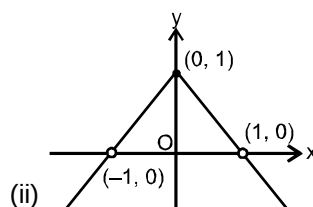
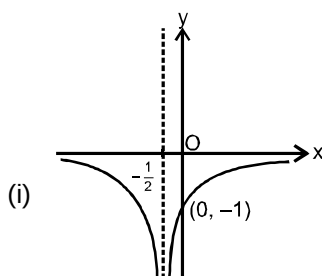
- D-1.** (i) $\left[\frac{1}{2}, 2\right) \cup (5, \infty)$ (ii) $[-1, (\sqrt{5}-1)/2)$ (iii) $x \in [3, \infty)$
 (iv) $x \in \left[\frac{7-\sqrt{21}}{2}, 2\right] \cup \left[4, \frac{7+\sqrt{21}}{2}\right]$ (v) $x = 2$
 (vi) $(-\infty, -5] \cup [1, \infty)$ (vii) $(-\infty, -3]$ (viii) $[-2, 0] \cup [\sqrt{2-\sqrt{3}}, \sqrt{2+\sqrt{3}}]$
 (ix) $(-1, 1) \cup (2, 3]$
- D-2.** $x = \log_2 a$ where, $a \in (0, 1]$

Section (E) :

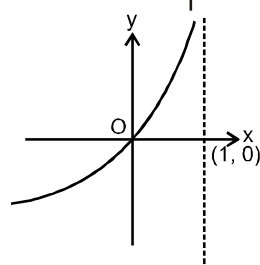




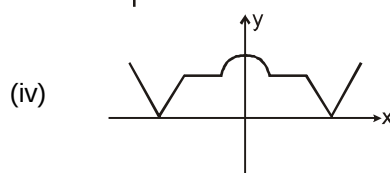
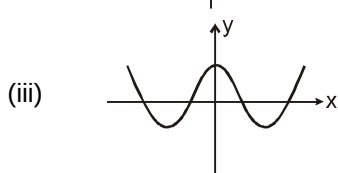
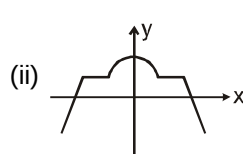
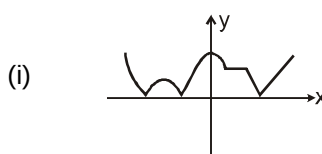
E-2.



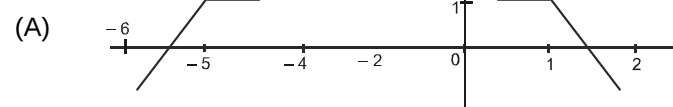
E-3.

E-4. $\lambda \in (12, 16)$

E-5.



E-6.



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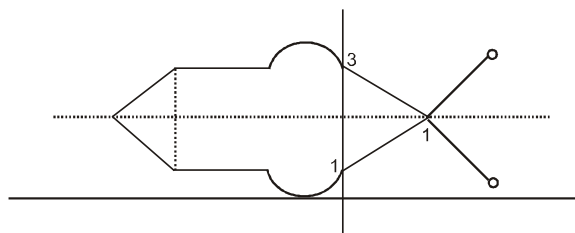
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(B)



E-7. (i) 2 (ii) 2

E-8 (i) $k \in \left(1, \frac{5}{4}\right)$ (ii) $k = 1, \frac{5}{4}$ (iii) $k = -1$ (iv) $k \in \left(\frac{5}{4}, \infty\right) \cup (-1, 1)$

EXERCISE # 2

- | | | | | | | |
|-----------|-----------|-----------|----------|---------|-----------|-----------|
| 1. (B) | 2. (B) | 3. (D) | 4. (A) | 5. (A) | 6. (A*) | 7. (C) |
| 8. (B) | 9. (A) | 10. (D) | 11. (C) | 12. (A) | 13. (D) | 14. (ABC) |
| 15. (ABC) | 16. (AB) | 17. (D) | 18. (B) | 19. (D) | 20. (A) | 21. (B) |
| 22. (C) | 23. (D) | 24. (BD) | 25. (AB) | 26. (D) | 27. (ACD) | |
| 28. (ACD) | 29. (ABC) | 30. (ABC) | 31. (C) | 32. (C) | 33. (A) | 34. (C) |

EXERCISE # 3

PART-I

- | | | | | |
|--------|--------|------------------------------|--------|--------------------------------|
| 2. (A) | 3. (B) | 4. (D) | 5. (C) | 6. $x = -1 - \sqrt{3}$ or -4 |
| 7. (A) | 8. (B) | 9. $\{-1\} \cup [1, \infty)$ | 10. 4 | 11. (B) 12. (D) |

PART-II

- | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|
| 1. (3) | 2. (3) | 3. (2) | 4. (4) | 5. (3) | 6. (2) | 7. (2) |
| 8. (1) | | | | | | |

